



Relative Efficiency and Price Discovery in the Two-Tier Brussels Stock Exchange

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Abstract

Until the mid-90s, the more active among the Brussels-listed stocks were traded in parallel segments: “spot” (that is, with $t + 3$ settlement) and “forward” (with periodic settlement at the end of a two- or three-week period). The forward market was cheaper, deeper, unhampered by price limits, convenient also for shortsales, and played by the pros; so it looks likely to be the more efficient tier. Yet a simple variance comparison, after correcting for settlement effects, finds no such effect; in fact, the forward market may actually have been the more noisy one.

Going on to the issue of price discovery, which is a more formal way of testing whether the forward market is less noisy, we extend the Margrabe-Silber price-discovery model to take into account the asynchronism in the opening forward and spot prices. Although the presence of the latent true morning return in our extended model precludes us from estimating explicitly the price adjustment coefficients, we can still identify the sign of the estimated difference of these coefficients and the lower and upper bounds of its t-statistic. This information enables us to conclude on the significance of the test. Also, our results reject the potential price discoverer status of the forward market: spot prices seem more informative than forward prices. This result raises the issue of how far the financial markets perform their central function of price discovery and how far the conventional wisdom can be trusted (e.g. the more trading volume the less noise the observed price contains).

JEL G14, G15

Key words: dual markets, price discovery, microstructure

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Introduction

Until 1996, the Brussels Stock Exchange had two parallel trading tiers: a “spot” market with third-day delivery where all stocks were traded and, for the most active stocks, also a parallel “forward” tier with fixed-date delivery. This paper addresses the issue as to which market was more noisy and which acted as the price discoverer, during the period 1989 - 1996. Both segments were order-driven, and their opening prices—the only ones for which we have a fairly precise idea about the timing—were set via a call. So there were no firm bids and asks, implying that the usual no-arbitrage predictions about price differences should be weakened into statements about expectations. The forward market was cheaper, deeper, unhampered by price limits, convenient also for shortsales, fully computerized, and played by the pros; so it looks likely to be the more efficient tier, with less noisy prices and doing most of the price-discovery work.

In an exploratory test we write spot and forward prices as equal to a common true value, plus a market-specific settlement effect (a correction for time value until payment), plus an unpredictable noise term. We expect to find that in the forward markets the noise has lower variance, that is, the pricing is more efficient. Yet we find no such thing; in fact, by some measures the forward market may actually have been the more noisy one.

Going on to the issue of price discovery, which is a more formal way of testing the price leadership hypothesis. From this angle, we perform the price-discovery test, market by market, using short-run price-discovery regressions, which is an extension of the price-discovery model of Margrabe and Silverman (1983). In our model, we take into account the asynchronism of the two opening prices; that is, where Margrabe and Silverman would measure the sensitivity of the price changes with respect to yesterday’s price discrepancy, we measure it with respect to yesterday’s forward premium plus the true morning return. The problem here is that the true morning return is unobserved, implying that the reaction speed coefficients of the two markets cannot be estimated explicitly. Fortunately, we can still identify the sign and of the estimated difference of the speed coefficients and the lower and upper bounds of its t-statistic. Against our expectations about the forward market being more efficient, the results from our

extended Margrabe-Silverman model reveal that, for more than half of the stocks, forward prices tend to be more influenced by yesterday's spot price than the other way around. Even more puzzlingly, the phenomenon is most pronounced for active stocks.

The rest of the paper is organized as follows. Section 1 describes the markets and the data. In Section 2 we present a standard noisy-price model and test whether forward prices appear to be the cleaner ones. Section 3 provides the tests for a price discoverer role of one segment. Section 4 concludes.

1 The Two-Tier Brussels Stock Exchange: Institutional Background

Brussels used to have not only its own stock market (the Brussel Stock Exchange (BSE), since 2001 integrated into Euronext), but even a two-tiered one: a “spot” market tier with third-day delivery, and for the most active stocks a parallel “forward” tier with fixed-date delivery. There used to be twenty-four fixed such settlement dates per year, implying that the trading periods typically lasted about two weeks—hence their name *quinzaine*, two-week period.¹ Details about the market organization are crucial for our analysis. In this section, we describe the price mechanisms in the forward and spot market and the delivery rules as they applied during the sample period.

1.1 The price mechanism in the forward tier

The forward market used to work via a pure public limit order book (which, during the sample period, was kept by a version of Toronto's Computer-Aided Trading System, CATS). Thus, although brokers were allowed to trade on their own account, they did not act as market makers, and their main role on the floor was to pass on the orders from the public to the exchange. At 9 p.m. the one-hour pre-market started, during which orders could be added or withdrawn and CATS displayed a continuously updated preliminary market-clearing price. Actual trading in the forward market started at 10 a.m., with a simultaneous call market for all stocks. That is, at 10 a.m. limit orders were matched as far as possible, and executed.

¹The forward market has now disappeared, following an EU-directed “ $T \leq t + 7$ days” rule implemented in the 1990s. London used to have a two-weekly fixed-delivery system too, Paris had delivery at the end of the month in its “forward” section for big stocks. (There also was a spot section for small stocks). Basel offered the choice between several delivery dates.

Table 1: Tick Size in the Spot and Forward Market

price range	price must be a multiple of	minimal percentage price change	
		at lower end of scale	at top end of scale
BEF 1-500	1	100%	0.20%
BEF 502-1,500	2	0.40%	0.13%
BEF 1,505-5,000	5	0.33%	0.10%
BEF 5,010-10,000	10	0.20%	0.10%
BEF 10,025-50,000	25	0.25%	0.05%
BEF 50,050	50	0.10%	—

Key One BEF is approximately EUR 0.025.

For most stocks the opening represented a substantial part of the day's turnover. After the opening round, the interactive trading session or "continuous market" started (10:00-16:30). Throughout the continuous-market session, the four best unfilled limit orders on the buying and selling side were displayed on computer screens and could be taken up by any incoming new order. Only brokers saw the screens: at the time of the sample, individual investors just heard (or saw) the opening and close prices over the radio or on Teletext, at noon or in the afternoon. Orders could also be matched directly, between brokers or in-house, provided that the price was within the book's bid-ask spread and the trade was reported immediately to the exchange. Large trades, *i.e.* blocks of at least BEF 50m (EUR 1,250,000) could be crossed or traded outside the BSE (often in London or Paris), but had also to be reported immediately. There were no limits on consecutive forward price changes. Limit order and trade prices were rounded according to a schedule shown in Table 1. Until the 1996 reform, the exchange's minimum margin requirement for a forward trade was 25 percent, but the BSE left the enforcement of this rule to the individual brokers (who bore the default risk). Securities could be posted as margin; in fact, many investors left most or all of their stocks with a their broker—most shares are bearer securities—and used this portfolio as margin for forward positions. Thus, there was no opportunity cost associated with the margin.

Prices for all traded lots were shown, in sequence (but not time-stamped), in the official price list, later *De Tijd* and *L'Echo de la Bourse*. In the electronic records, only open/close/high/low are available.

1.2 The Spot Price Mechanism

Due to its lower volume, the spot market was fully computerized much later (in 1996). Like the forward tier, it was order-driven, but the implementation was fairly different. First, there was no pre-market, so that the opening price was potentially much more subject to noise than the forward opening price even apart from volume effects. Second, because of the thinness of the market, for most stocks there was just one trading round per day. A continuous market existed only for the more active stocks (quoted on the “*corbeille*” segment), and even this continuous market was not very active.² Third, there was no centralized public order book kept by the exchange. Rather, a few specialist brokers each kept their own books, and met sometime between 1 and 1.30 p.m. on the Exchange’s floor to aggregate their information and identify the price that maximizes trade from the combined order book. Fourth, for stocks that were not traded on the parallel forward market there were overnight price limits of 5 percent (for very thinly traded stocks, traded on the *parquet* segment) or 10 percent (for other stocks, traded on the “*corbeille*” market); and, in the *corbeille* market, subsequent intraday price changes could not exceed 5 percent.

The actual pricing and trading was organized by a BSE official, who started by crying out a price proposal. This price proposal equaled the price that maximized trade from the order book if that price was within the price change limits; if not, the official announced the price limit itself. In addition to the price proposal, the official also announced the direction of the imbalance. If there was an excess supply (demand) at the proposed price, additional purchase (sale) orders from the floor were solicited to reduce the imbalance in the book. If the remaining imbalance between supply and demand at the price limit was less than 50 percent, the specialist would decide to ‘reduce’ most or all orders on the excess side, *i.e.* execute only part of each order; the transaction price was then published in the financial press with the qualification “*sellers reduced*” or “*buyers reduced*”. If at the price limit the imbalance between supply and demand remained huge even after soliciting orders from the floor, there was no trade at all, and the price limit was published as an indicative price. In practice, however, when the imbalance was only slightly larger than 50 percent the stock’s specialist brokers often added buy or sell orders for their own account to prevent no-trade (and no-commission-income)

²For stocks quoted on the *corbeille*, the fixing was followed by the traditional (blackboard-and-chalk) version of the continuous market: unfilled orders were chalked onto the blackboard and could be picked up from the floor, and orders could also be matched directly on the floor at a price within the book’s spread.

days.

As, around 1990, the spot market list contained about 300 stocks, the stock-by-stock opening-call prices were set more or less sequentially; the exact timing of each stock's spot fixing was not registered. For the *corbeille* market, prices for all traded lots were shown, in sequence (but not time-stamped), in the official price list but in the electronic records, only open/close/high/low are available. For the *parket* stocks there is just the single price.

1.3 Settlement Rules

For the BSE, the other details of the actual settlement were similar for both market tiers. The buyer paid via a bank transfer rather than by check. This means that there was no “mail float” on the payment side. Still, the value dates for buyer and seller did not match perfectly: the buyer's value date is one day before the actual settlement day, and the seller obtains value one day after settlement.

Delivery of the stock could mean actual physical delivery of the piece of paper that represents the bearer share, if the buyer desired so. Alternatively, the buyer could ask that his or her purchase be recorded with a netting and depository institution, the *Caisse Interprofessionnelle/Interprofessionale Kas* (CIK). The CIK merely netted the physical deliveries across brokers if actual delivery was asked, and held the paper on behalf of investors that did not demand physical delivery. Thus, the CIK was not a clearing house in the usual sense: it did not act as a central counterpart, nor did it cancel an individual investor's earlier purchases against subsequent sales (or *vice versa*) within one settlement period. There was some informal clearing by brokers, though: brokers did not exact delivery and payment for a forward transaction that was reversed later on via the same brokerage house and within the same quinzaine.

One function of the forward market, therefore, was to reduce the cost and hassle of mutually offsetting stock deliveries and payments for trades that had been closed out within the same quinzaine. This partly explains why, unlike in currency markets, in the forward tier the transaction costs for small trades were somewhat lower than in the spot tier (as illustrated in Table 2).³ A second useful feature of the forward tier is that it allows one to take short positions until the end of the quinzaine, positions that could then be rolled over fairly easily.

³Another reason for the lower transaction costs might have been the fact that the forward market tended to have larger volumes than the spot market for the same stock.

Table 2: **Transaction Costs, Spot and Forward, 1990**

item	cost of spot trades	cost of forward trades
BSE Commission	0.03%, max BEF 6 000†	
Transaction Tax	0.17%, max BEF 10 000	
Brokerage fees		
- fixed part	BEF 200*	
- variable part:		
order BEF 1-5m	1%	.8%
order BEF 5-10m	.8%	.6%
order BEF 10-20m	.4%	.3%
order BEF 20-30m	≥ BEF 130 000‡	.2%
order ≥ BEF 30m	≥ BEF 130 000‡	≥ BEF 120 000‡

† : 40 BEF is worth approx. 1 EUR; * : plus BEF 100 for the buyer if physical delivery is asked; ‡ : negotiable, with the stated amounts as minima. Thus, a smallish trade of BEF 250,000 (approx. EUR 6.250) would cost 1.29 percent spot, and 1.09 percent forward. For an order of BEF 30m, the cost difference may be as small as $10,000/30,000,000 = .033$ percent.

In Belgium, there was no formal legal framework for asset borrowing and spot short-selling until the 1991 Stock Market Reform Act, so until then the forward market provided the sole organized opportunity for short positions. A third function of the forward market was to provide the equivalent of buying on margin: the actual payment was deferred until the end of the *quinzaine* (at which moment the forward contract could be rolled over), and the buyer just posted the 25 percent security. Since leveraged buying was possible in the forward market, no organized system of buying on margin was set up in the spot market.

1.4 Possible clientele and differential information aspects

It is fair to say that the organization of the forward markets was superior: it was fully computerized by the late 80s, had a pre-market, enjoyed lower costs and no price limits, and was much deeper. In addition (or, perhaps, as a result of the above), conventional wisdom within the financial community held that there also was an clientele- and efficiency-related form of segmentation.⁴ Indeed, because of its shorting facilities and the absence of price limits, the forward market had a somewhat more speculative reputation, to the extent that conservative firms (such as the major banks) have long resisted a forward listing. Because of this speculative image, the forward market was considered to be the market for the more professional agents, while less sophisticated investors were said to prefer the spot market. Having no systematic

⁴We are indebted to the late Prof. Van Essche for this suggestion.

and fast access to news during working hours, these amateur traders allegedly reacted more slowly than the professionals. In the terminology of Garbade and Silber (1983), this view hypothesizes that the forward market was the price discoverer, while the spot market was just a (lagging) satellite market. This hypothesis is the central issue of the paper.

1.5 Taxes

A last relevant detail is income tax. For brokers or corporations, all interest received or paid and all short-term capital gains or losses are fully taxable or deductible. So if brokers or corporations dominate the market in the sense that they are systematically the marginal traders, taxes are neutral. Under personal taxation rules, capital gains or losses are neither taxable nor deductible; nor can one deduct interest costs incurred to finance short-term trades; and interest income is *de facto* taxed at the withholding tax (10 percent at the time). In short, also for private persons the gross rate is the relevant interest rate, unless the marginal traders are buyers of stock for whom the opportunity cost is the interest foregone on a deposit.

Dividends are largely tax-exempt for corporations; for individuals, a 25-percent withholding tax applies. Unpublished tests show that the average price drop on ex-dividend day was equal to the dividend net of the withholding tax—20 percent before 1984, and 25 percent thereafter. All returns are accordingly computed from dividends net of withholding taxes.

We conclude the descriptive section with some information on the data.

1.6 Data

The sample period starts early 1989, at which time the forward markets was fully computerized, and ends in 1996; in 1997 the forward market disappeared. Euronext’s historic-data CDs for that period include the opening spot price per day, and, for the forward market, the daily opening, high and low, and close price. Data on dividends, bonus dividends, splits, and rights issues⁵ were missing, and were hand-collected from *Memento der Effecten*, a trade publication, and from *De Tijd*, which published the Dutch-language version of the Official Price List. For the risk-free rate we used the Euro-BEF 1 week middle rate from Datastream.

We discarded foreign stocks, about half of the list, since price discovery for these shares

⁵A subscription right is represented by a coupon designated for the purpose, and is traded separately the moment the stock goes ex this coupon. The market values of these “scripts” are very noisy so we worked with the standard intrinsic value of a subscription right.

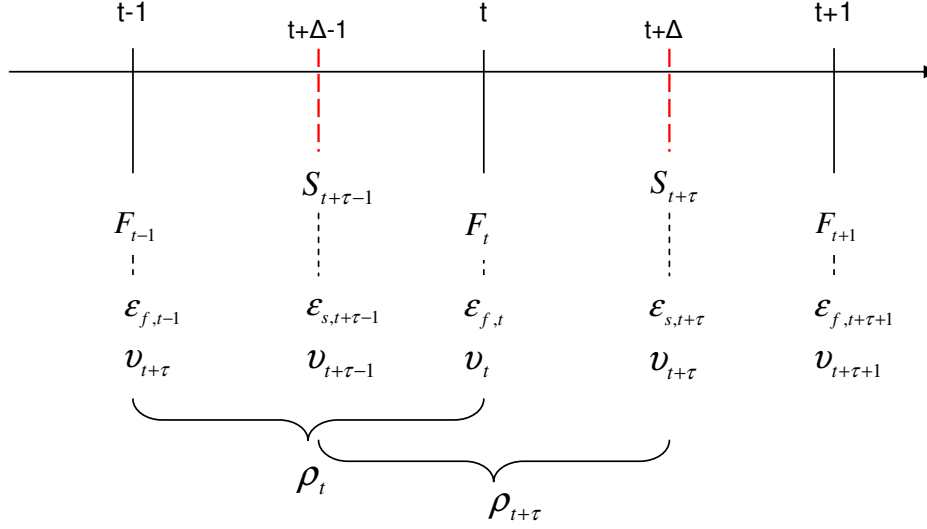
Table 3: **Trading Frequency and One-day Return Variance across Turnover Classes**

sample (by turnover)	number of returns		Average Variance		Median Variance		numbers of stocks $\text{var}_s > \text{var}_f$
	Spot	Forward	Spot	Forward	Spot	Forward	
All	95,591	87,549	3.26	3.43	2.23	1.91	37 (out of 72)
low turnover	27,576	21,686	4.55	5.17	2.77	2.11	14 (out of 24)
medium turnover	31,169	29,197	3.23	3.12	1.92	1.66	11 (out of 24)
high turnover	36,848	36,670	1.99	2.00	1.61	1.88	12 (out of 24)

Key Each turnover class contains 24 stocks, and ranking is done on the basis of average daily turnover. Variance is computed for the percentage daily returns—that is, in units E-4 for regular per-unit.

probably happens abroad anyway. So we started from data on 119 Belgian stocks traded on both the spot and forward tiers of the Brussels Stock Exchange during the period 1989-1996. Some data cleaning was required: 16 stocks are excluded due to an insufficient number of observations (too many missing data points), 31 stocks are connected to other shares due to a change in the name or code after a stock split or merger. Thus, 72 stocks remained. All unusually large forward premia or large change in the prices were double-checked with the prices posted on the hard copies of *De Tijd*, including the next-day rectifications for typos. All prices that are indicated ‘sellers reduced’, ‘buyers reduced’, or ‘indicative’, were considered to be missing observations. Whenever there is a missing price, the two returns that are associated with that price are missing too. That is, we never use cumulated returns straddling some missing price.

Eight years of data means over 2000 trading days. The number of effectively available observations is very variable, ranging from below 50% to 100%. There is a clear relation with market activity. As can be seen in Table 3, the firms in the bottom third, by turnover, on average trade only 75 (spot) or 60 (forward) percent of the time. Forward markets more often have missing prices than spot markets, despite their higher turnovers and the absence of price limits; this probably reflects the interventions by the spot market’s specialists mentioned in Section 1.2. There also is a strong negative relation between turnover and return variance, *prima facie*, as also illustrated via Figure 2. Much of that, however, seems to be due to the outliers: when we consider medians the schedule is much flatter. In terms of numbers of stocks whose spot variance exceeds the forward variance, the tally is almost perfectly even.

Figure 1: **Asynchronism of Spot vs Forward Prices**

Key t is at 10:00 am and $t + \tau$ is at 13:30 p.m.

2 An Exploratory Test of Relative Informational Efficiency

2.1 The Test

Let v_t denote an unobservable true value based on full and correct use of all relevant available information, expressed as a price for immediate payment and delivery. Since neither the actual spot nor forward prices imply immediate settlement, the corresponding true “spot” and forward values, denoted as s and f , should contain a settlement effect shown below, with n_s and n_f denoting the number of calendar days to settlement and R the simple per diem interest rate. In addition, actually observed prices are assumed to deviate from true values by a zero-mean, i.i.d. noise term, denoted by ϵ_s or ϵ_f , respectively, which reflects unanticipated orders by liquidity traders and noise traders, as standard in microstructure models⁶:

$$s_{t+\tau} = (1 + n_{s,t}R_t)v_{t+\tau}, \quad (1)$$

$$f_t = (1 + n_{f,t}R_t)v_t, \quad (2)$$

$$S_{t+\tau} = s_{t+\tau}(1 + \epsilon_{s,t+\tau}) = v_{t+\tau}(1 + n_{s,t}R_t)(1 + \epsilon_{s,t+\tau}), \quad (3)$$

$$F_t = f_t(1 + \epsilon_{f,t}) = v_t(1 + n_{f,t}R_t)(1 + \epsilon_{f,t}). \quad (4)$$

⁶We ignore the time value of half a day as interest is calculated per entire day only.

where time t is 10:00 a.m., the opening of the forward market; time $t + \tau$ is 13:00 p.m, the opening time of the spot market. In an efficient but noisy market, for any $\Delta > 0$ we expect $E_{t-\Delta}(\epsilon_{s,t+\tau}) = 0 = E_{t-\Delta}(\epsilon_{f,t})$. By postulating that both spot and forward prices are just noisy versions of the same true value process we clearly assume perfect integration of the two markets apart from their order books.

These models are not ready for use in itself as they contain unobservable prices. The standard way to make such models tractable, in the sense of being able to identify some key parameters, is to consider returns—percentage changes in S or F , as is done below. In (5) and (7), the true values have been combined into a true return, denoted as ρ_t , which is then assumed to be unpredictable white noise; we also introduce the shorthand notation $\Delta n_s R$ and $\Delta n_f R$ to indicate the settlement effect in a spot or forward returns, and e to indicated $\ln(1 + \epsilon)$. Thus, for the spot market we have

$$\begin{aligned} r_{s,t+\tau} &:= \ln \left(\frac{S_{t+\tau}}{S_{t+\tau-1}} \right), \\ &= \ln \left(\frac{1 + n_{s,t} R_t}{1 + n_{s,t-1} R_{t-1}} \right) + \ln \left(\frac{v_{t+\tau}}{v_{t+\tau-1}} \right) + \ln(1 + \epsilon_{s,t+\tau}) - \ln(1 + \epsilon_{s,t+\tau-1}), \end{aligned} \quad (5)$$

$$=: \Delta(n_s R)_t + \rho_{t+\tau} + e_{s,t+\tau} - e_{s,t+\tau-1}, \quad (6)$$

and likewise, in the forward tier,

$$\begin{aligned} r_{f,t} &:= \ln \left(\frac{F_t}{F_{t-1}} \right), \\ &= \ln \left(\frac{1 + n_{f,t} R_t}{1 + n_{f,t-1} R_{t-1}} \right) + \ln \left(\frac{v_t}{v_{t-1}} \right) + \ln(1 + \epsilon_{f,t}) - \ln(1 + \epsilon_{f,t-1}), \end{aligned} \quad (7)$$

$$=: \Delta(n_f R)_t + \rho_t + e_{f,t} - e_{f,t-1}. \quad (8)$$

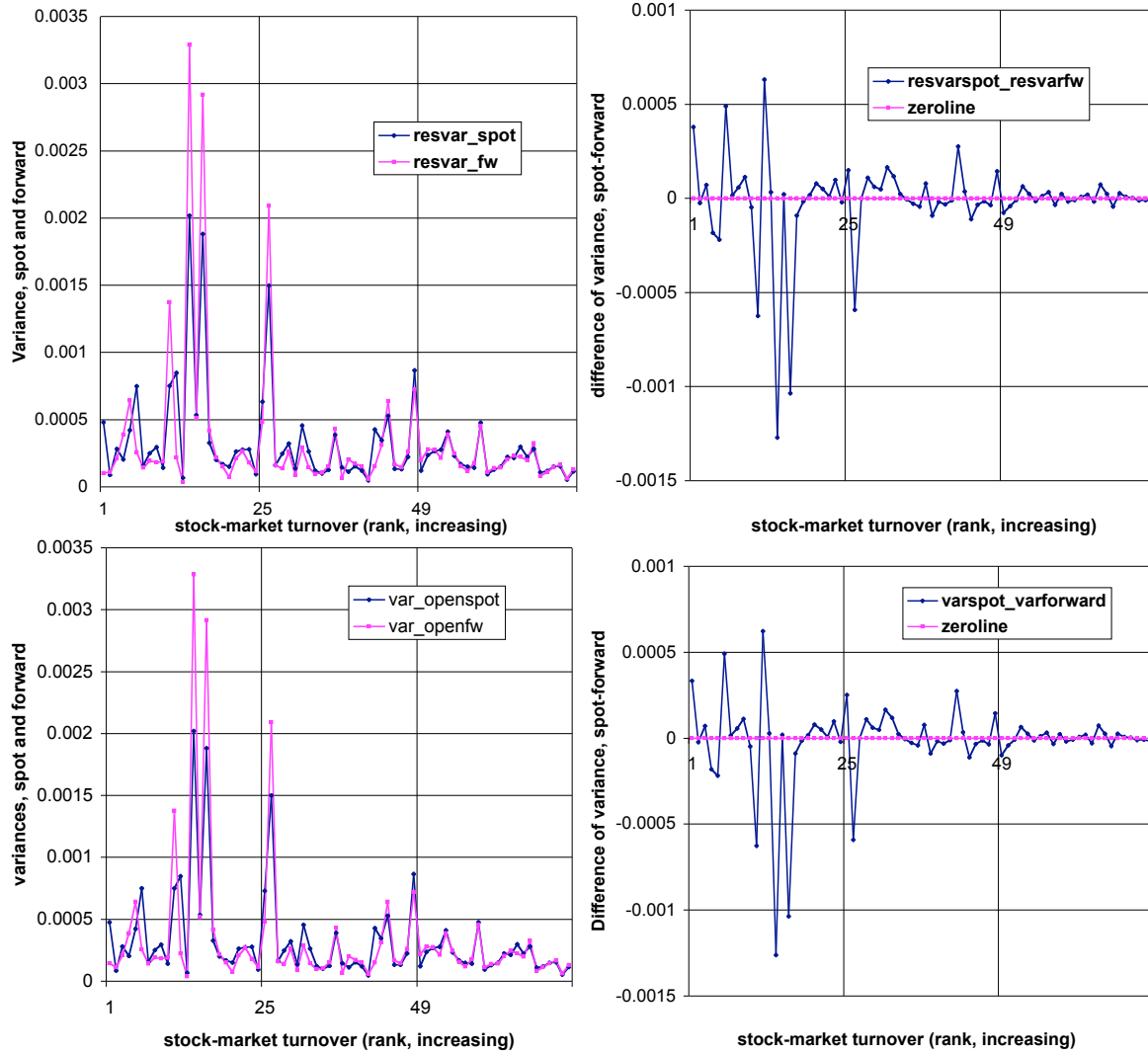
This immediately induces a simple variance measure for the relative noisiness of the two segments:

$$\text{var}(r_{s,t+\tau} - \Delta n_s R_t) = \text{var}(\rho_{t+\tau}) + 2\text{var}(e_{s,t}), \quad (9)$$

$$\text{var}(r_{f,t} - \Delta n_f R_t) = \text{var}(\rho_t) + 2\text{var}(e_{f,t}). \quad (10)$$

If true returns are serially independent, the variance of an open-to-open return (ρ_t) is the same as the variance of a noon-to-noon return ($\rho_{t+\tau}$). Equations (9) and (10) then unsurprisingly say that a straightforward comparison of time-value-adjusted return variances should tell us which market is the more noisy one.

Figure 2: Variances of daily returns, spot v forward; opening prices, all days



Key Daily returns are computed iff there were transactions on the day and the preceding trading day, regardless of the situation in other market segment. Variances are computed from returns corrected for time to maturity (top) or from raw returns (bottom). The graphs on the left-hand side show the two variances; the ones on the right show the difference, spot minus forward.

2.2 Results

Table 3, above, already provides some results on variances across markets, but the picture is far from clear. In terms of simple averages, the spot market seems to be relatively more efficient while in terms of medians the picture is exactly the opposite. This holds for the all-stock averages and the low- and high-turnover groups classes. The rightmost panel of Figure 2 displays the differences (spot variance minus forward variance) stock by stock, with stocks

Table 4: **Spot v Forward Return Variances: Aggregates**

	Returns corrected for time value	
	Spot	Forward
All	2.848	3.058
Low turnover	3.761	4.600
Medium turnover	3.050	3.234
High turnover	1.993	2.006

Key Variances of percentage daily returns—that is, in units E-4 for regular per-unit—are estimated as nobs-weighted averages of the series-by-series estimates. The turnover-based subsamples each contain 24 stocks.

arranged by increasing turnover rate;⁷ among the cases with big differences, there are more negative ones, that is, more cases where the forward market does more poorly.

Significance tests are hampered by the fact that a stock's sample variance in the spot market cannot reasonably be assumed to be independent of that stock's sample variance in the forward market. Another problem is that much of the counterintuitive results for average variances may be due to the numbers of observations per series, which are higher for big-turnover stocks but also for spot returns. As a straightforward correction for the number of observations, we provide an average variance weighted by the number of observations per series, N_j :

$$\overline{\text{var}} = \sum_j \frac{N_j \text{var}_j}{\sum_j N_j}. \quad (11)$$

This is similar to first demeaning, stock by stock, and then pooling or stacking all squared demeaned returns, thus giving all observations equal weight, rather than assigning each stock's estimated variance the same impact regardless of the number of observations underlying the estimate.

Table 4 shows the results for the aggregate variances. Relative to the numbers in Table 3, all average variances are down, as expected when noisy estimates get less weight. Second, both for the total sample and for the three size subsamples the spot market continues to have lower variance. This remains counterintuitive, and at variance with a ranking on the basis of medians (Table 3).

⁷In the sections that follow we often use graphs to display results for the 72 stocks. For that purpose, stock are always ranked on the basis of turnover, as we did in Table 3, from low to high, and points relating to the same variable—say, spot variance—are connected by a line. This makes them look like time series plots, but that is not the proper interpretation.

Table 5: **Variances of Daily Returns, Spot v Forward; Close Prices, All Days**

sample (by turnover)	Average Variance		Median Variance		Number of stocks with $\text{var}_s > \text{var}_f$
	Spot	Forward	Spot	Forward	
All	2.99	3.33	1.73	1.91	22 (out of 52)
low turnover	3.20	5.18	2.39	2.18	7 (out of 15)
medium turnover	4.75	2.86	3.09	1.70	10 (out of 16)
high turnover	1.40	1.95	1.36	1.72	5 (out of 21)

Key Each turnover class contains 24 stocks, and ranking is done on the basis of average daily turnover.

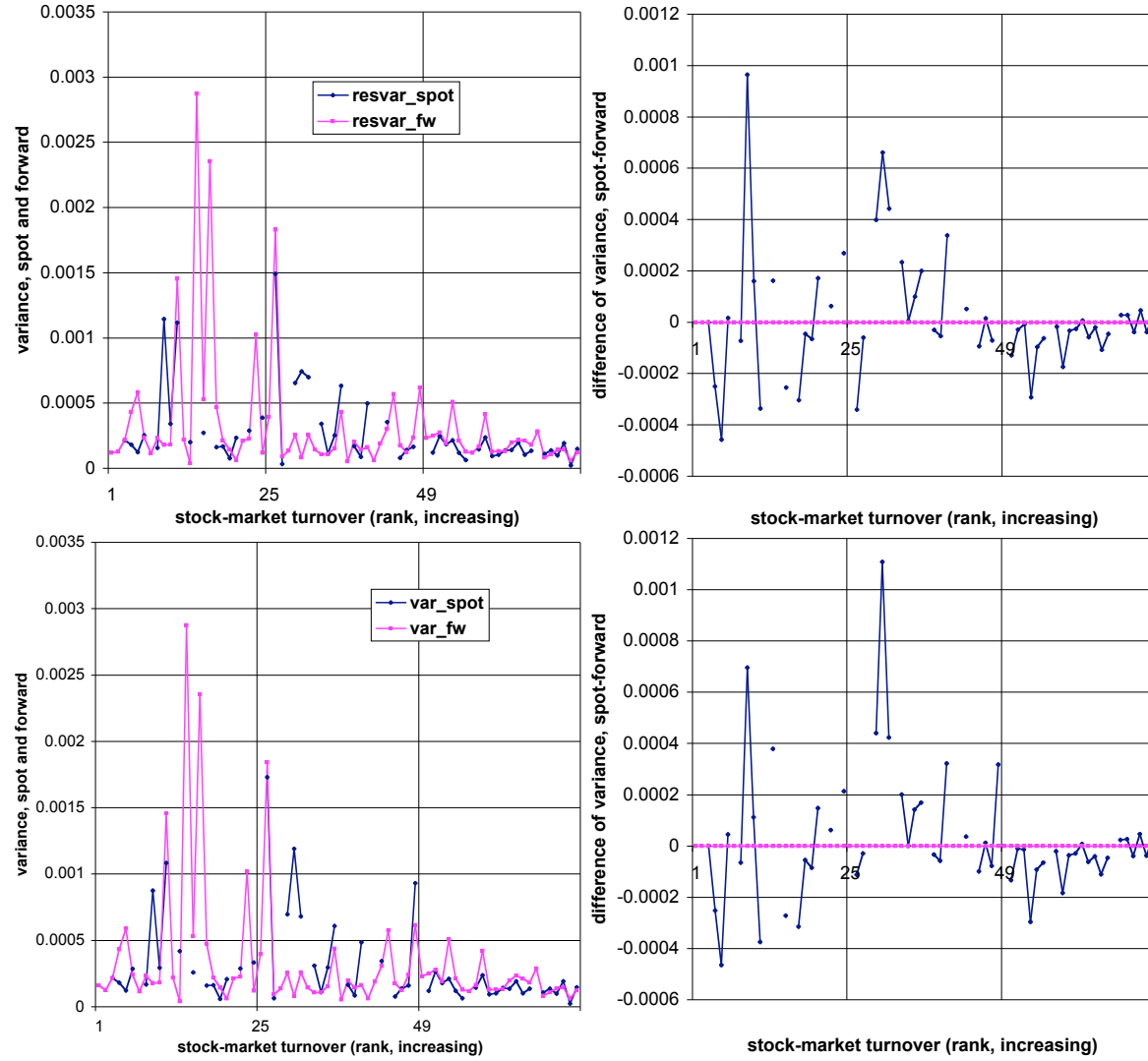
2.3 Robustness Tests

There are a number of issues with the first-pass tests that need to be looked into. A first potential problem is that, while both prices are, formally, opening prices, the spot price is still set after a few hours of forward trading, so it is economically speaking not an opening price in the sense commonly employed in the market-microstructure theory. This matters because an opening price is viewed as systematically more noisy than an intraday price. Perhaps this is unlikely to be a major factor in the explanation of our puzzle: even the forward price is not really an opening price, as it follows one hour of pre-trading; this gives players ample time to see the consensus price evolve and to modify orders or look for extra information. Still, we test the robustness by looking at the prices at the close instead of the open. Last-trade spot and forward prices are closer together, timewise, and reverse any systematic information advantage that might be present in spot opening prices. The cost is that this only works for the *corbeille* stocks: we lose about one-third of the sample, namely, the stocks for which there was just one daily fixing.

The results, in Figure 3, are not very different but the spot market comes out as the less noisy one more often. Across all stocks, the spot variance is lower in terms of mean, median, and number of stock-by-stock comparisons (22 out of 52). While the lead is fairly strong in itself, it is even more remarkable in light of the organizational advantages the forward market enjoyed. Equally unexpected, the lead is particularly pronounced among the heavily traded stocks, group 3.

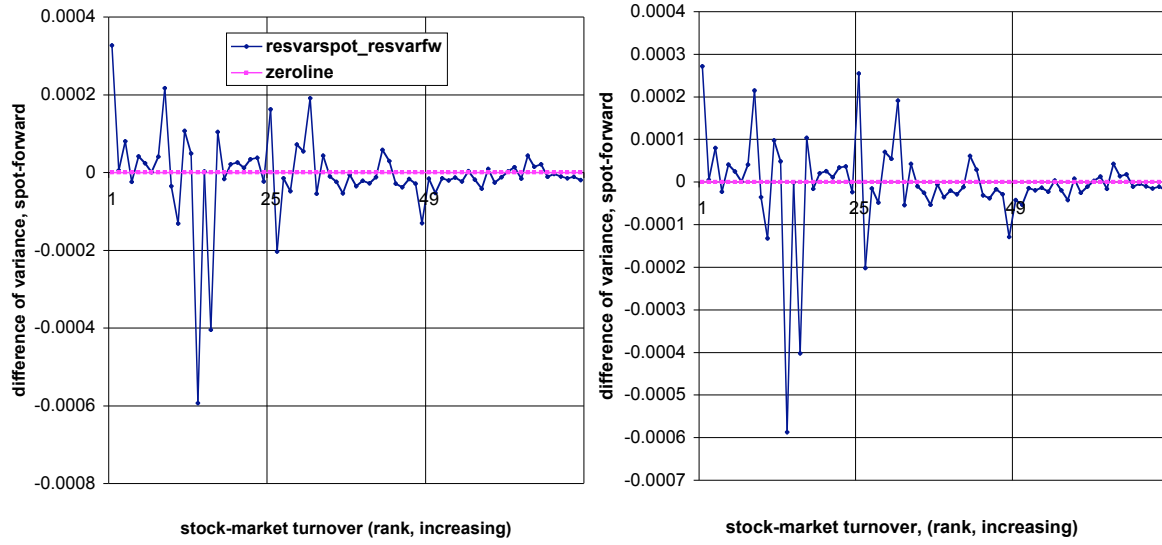
We next consider potential biases due to different samples. In the first-pass tests, the variances were computed from all available one-day returns in that market segment, that is, all days where there was a price on that day and on the preceding trading day. The no-trade

Figure 3: Variances of daily returns, spot v forward; close prices, all days



Key Daily returns are computed iff there were transactions on the day and the preceding trading day, regardless of the situation in other market segment. Variances are computed from returns corrected for time to maturity (top) or from raw returns (bottom). The graphs on the left-hand side show the two variances; the ones on the right show the difference, spot minus forward. For about one third of the stocks the spot market had just one daily fixing; for these stocks no variance is computed and, of course, no difference of variances is shown either.

days on spot and forward markets do not coincide, though. Days with no trade in one tier may also be thin-trading days (with, presumably, more noisy returns) in the other section, which would then bias the variance upward in the tier with most trading days. Figure 4 and Table 6 shows the results for time-value-corrected and raw returns, in which the spot and forward samples are matched. The picture is again more clearly in favor of a lower variance for spot returns. The biggest differences still occur for the low-turnover group of stocks, even though in that segment the forward market leads in terms of pure numbers (spot variance is smaller

Figure 4: **Variances of daily returns, spot v forward; opening prices, matched sample**

Key Daily returns are computed iff there were transactions on the day and the preceding trading day in both spot and forward markets. Variances are computed from returns corrected for time to maturity (left) or from raw returns (right). The graph shows the difference of the variances, spot minus forward.

Table 6: **Variances of Daily Returns, Spot v Forward; Opening Prices, Matched Sample**

sample (by turnover)	Average Variance		Median Variance		Number of stocks with $\text{var}_s > \text{var}_f$
	Spot	Forward	Spot	Forward	
All	3.14	3.21	2.09	1.93	30 (out of 72)
low turnover	4.60	4.66	2.96	2.11	16 (out of 24)
medium turnover	2.88	2.90	1.44	1.67	7 (out of 24)
high turnover	1.96	2.06	1.76	1.97	7 (out of 24)

Key Each turnover class contains 24 stocks, and ranking is done on the basis of average daily turnover.

only 8 times out of 24). For the medium- and high-turnover groups, the spot market is less volatile 17 times out of 24, meaning that in total the spot market still leads in 42 cases out of 72. Means and medians confirm that picture. This makes sense given that the spot market actually is the tier with fewer no-trade days: what we find is that if we discard days where the forward market does not trade, then the variance in the spot market for the remaining days is relatively lower.

There are two broad conclusions from this section. First and foremost, there is little evidence that the forward market, which seemed to hold all the good cards, leads in terms of price discovery; in fact, what evidence there is tends to lean the other way. A second, more

technical, fact is that the time value corrections made very little difference.

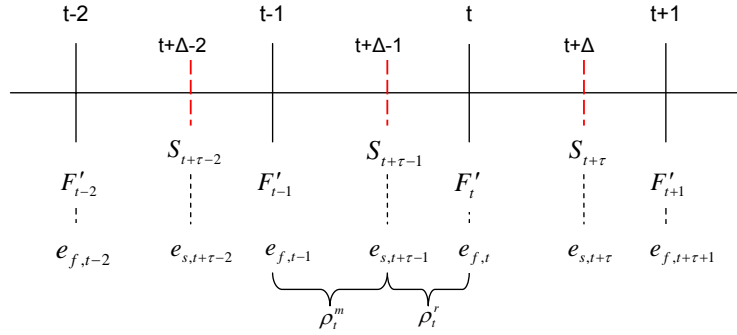
We now have a closer look at the price-discovery issue, which is a more formal way of examining whether the forward market is less noisy. First of all, looking at the efficiency issue from this angle allows us to obtain a formal test and its t-statistics for the difference of noise variances. Secondly, we can relax the assumption of no autocorrelation in the true returns when comparing the variances, which is in line with the significant autocorrelation of the forward premium we found in the data. Thirdly, we show theoretically that the difference of the noise variances is proportional to the difference of the price adjustment coefficients in the price-discovery regression. Last but not least, this approach enables us to understand the mechanism of price adjustment. As price-discovery tests use regressions involving both spot and forward returns, the sample used in the rest of the paper is the ‘matched’ one as used in the third variant of the exploratory tests.

3 Price-Discovery Test

Let us define F' as the forward price corrected for settlement-time differences relative to the spot market:

$$F'_t := F_t \frac{1 + n_{s,t}R_t}{1 + n_{f,t}R_t}. \quad (12)$$

Figure 5: **Spot and Forward prices**



We say that the two market tiers are well integrated if, for any positive forecasting horizon Δ , the expectations about simultaneous spot and forward prices are identical:

$$E_{t-\Delta}(S_t) = E_{t-\Delta}(F'_t). \quad (13)$$

In terms of our earlier noisy-price equations, a sufficient assumption is that both markets use a common true price v and that pricing errors have zero expectation. Equation (13) is the

closest one can get to the usual Law of One Price if prices are set by a call rather than quoted by market makers. This equation could have been utilized if S_t , the spot price at 10 a.m, had been observable. In reality, our forward prices are observed at 10:00 o'clock while spot prices are from 13:00-13:30. If markets are well integrated, then the *ex post* forward premium or price discrepancy, $\ln F'/S$, should consist of the negative of the 10:00-13:00 return plus two pricing errors. To keep track of all this, let us introduce a more precise notation. The percentage change of the true value, 10:00 to 10:00, is split into a morning part (10:00 to 13:00) and a rest-of-the-day part (13:00 to 10:00 next day), denoted as $\rho_t = \rho_t^m + \rho_t^r$. The observed returns and premia can now be decomposed as follows. In the forward return, the true-value price change is split in two parts; so is the spot return; and the forward premium is now recognized as having a 10:00-to-13:00 true-return component:

$$r_{f,t} := \ln F'_t/F'_{t-1} = (\rho_t^m + \rho_t^r) + e_{f,t} - e_{f,t-1}, \quad (14)$$

$$r_{s,t+\tau} := \ln S_{t+\tau}/S_{t+\tau-1} = (\rho_t^r + \rho_{t+1}^m) + e_{s,t+\tau} - e_{s,t+\tau-1}, \quad (15)$$

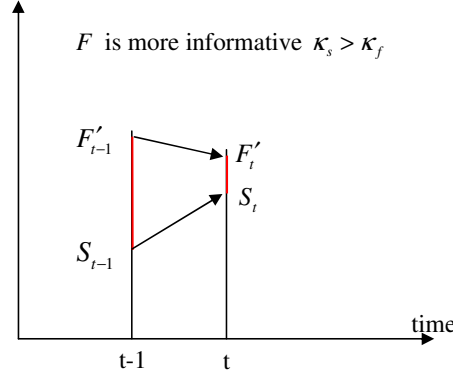
$$p_{t-1} := \ln F'_{t-1}/S_{t+\tau-1} = e_{f,t-1} - (\rho_t^m + e_{s,t-1+\tau}). \quad (16)$$

In order to develop a model to test for the price-discovery status of the forward market, we start from a well-integrated and efficient market, as in Equation (13). This means that the conditional expectations of the microstructure noise terms are zero, implying that the e s are not auto- or cross-correlated. Still, we do observe autocorrelation in both the returns and the forward premia; the only explanation left, then, is that this autocorrelation is present in the true returns, reflecting fluctuations in risk like GARCH effects or earnings announcements and other events. We also allow the noise terms in the same day to be correlated: even though the spot price is set later than the forward price, orders for both markets are likely to have been submitted at the same time, in the morning, and few of them may have been subsequently modified in light of the forward opening price.

The above implies the following assumptions, to which we add the earlier assumption that the variance of the true morning-to-morning and noon-to-noon returns are the same:

- (i) The noise has zero auto- and cross-correlation in the sense that $\text{cov}(e_{f,t}, X_{t-\Delta}) = 0 = \text{cov}(e_{s,t+\tau}, X_{t-\Delta})$; only $\text{cov}(e_{s,t+\tau}, e_{f,t})$ may be nonzero.
- (ii) There may be first order autocorrelation of true returns.
- (iii) The covariance of the today's true morning return and the true rest-of-the-day return equals to the covariance of the today's true morning return and the true yesterday's

Figure 6: Measuring Price Leadership



rest-of-the-day return: $\text{cov}(\rho_{t-1}^r, \rho_t^m) = \text{cov}(\rho_t^m, \rho_t^r)$. Accordingly, the same true one-day return variance is implied in both markets.

In the context of asynchronous prices, the hypothesis of market integration in Equation (13) must be modified into the statement that the expected forward premium plus the true morning return conditional on all information available right before the markets opens is zero:

$$E_{t-\Delta}(\ln F_t - \ln S_{t+\tau} + \rho_{t+1}^m) = 0. \quad (17)$$

Equation (17) is utilized for a price-discovery test in the next subsection.

3.1 The Test Equations

The integrated-market hypothesis in Equation (17) states that the expected forward premium plus the true morning return, conditional on all information available right before the markets open, is zero. This means that, whatever yesterday's forward premium plus the true morning return turned out to be, on average all of it should be gone the next day.

A weaker version of the integration property would state that abnormal forward premium plus the true morning return tend to revert towards zero, but perhaps need more than one day to achieve this. Whether reversion is complete or not, after one day, the second question is which market does most of the adjustment, that is, which is more of a follower instead of a leader. If the true morning return ρ_t^m were observed, the leadership could have been identified

by running two simple error-correction models (ECMs):

$$E_{t-\Delta}(\ln F'_t - \ln F'_{t-1} | S_{t+\tau-1}, F'_{t-1}) = \alpha_f - \kappa_f (\ln F'_{t-1} - \ln S_{t+\tau-1} + \rho_t^m), \quad (18)$$

$$E_{t-\Delta}(\ln S_{t+\tau} - \ln S_{t+\tau-1} | S_{t+\tau-1}, F'_{t-1}) = \alpha_s + \kappa_s (\ln F'_{t-1} - \ln S_{t+\tau-1} + \rho_t^m). \quad (19)$$

If $\kappa_s > \kappa_f$, then S is more of a follower than a leader, and vice versa.

In the case of synchronism, where $\tau = 0$ and $\rho_t^m = 0$, these ECMs reduce to the price-discovery equations by Garbade and Silber (1983):

$$E_{t-\Delta}(\ln F'_t - \ln F'_{t-1} | S_t, F'_{t-1}) = \alpha_f - \kappa_f (\ln F'_{t-1} - \ln S_{t-1}), \quad (20)$$

$$E_{t-\Delta}(\ln S_t - \ln S_{t-1} | S_{t-1}, F'_{t-1}) = \alpha_s + \kappa_s (\ln F'_{t-1} - \ln S_{t-1}). \quad (21)$$

Their derivation is based on mean-variance optimizing agents that have access to a spot and futures market for one asset only. Figure 6 depicts the error correction mechanism of the prices in this synchronism context.

Returning to the ECMs in our asynchronism case, if the true morning return ρ_t^m had been observed, the slopes in Equations (18) and (19) could have been estimated explicitly. Our objective is to estimate the κ s, or at least their difference, even when ρ_t^m is latent. Moreover, we also want to test for the significance of the estimated differences. To this end, we add up both sides of the ECMs. As a result, we get a regression of the sum of the spot and forward returns on the initial forward-premium plus the true morning return, with a slope whose sign equals $\kappa_s - \kappa_f$. Thus, a positive slope in the sum-of-return regression tells us that $\kappa_s > \kappa_f$, *i.e.* the spot market is more of a follower:

$$\begin{aligned} E_{t-\Delta}[r_{f,t} + r_{s,t+\tau}] &= (\alpha_f + \alpha_s) + (\kappa_s - \kappa_f)(p_{t-1} + \rho_t^m), \\ &= \delta_0 + \delta_1(p_{t-1} + \rho_t^m). \end{aligned} \quad (22)$$

For convenience, let us denote

$$\begin{aligned} y_t &= r_{f,t} + r_{s,t+\tau}, \\ x_t &= (p_{t-1} + \rho_t^m), \end{aligned}$$

which simplifies Equation (22) to:

$$y_t = \delta_0 + \delta_1 x_t + \xi_t. \quad (23)$$

Since we assume the true return is first-order autocorrelated (Assumption (ii)), so may be the error term ξ_t . Let h be the first-order autocorrelation coefficient of the error term, which is

estimated as:

$$h = \frac{\text{cov}(\xi_t, \xi_{t-1})}{\text{var}(\xi_t)}. \quad (24)$$

The following transformed model is homoskedastic non-autocorrelated, and thus the OLS estimate of the slope is efficient⁸.

$$y_t - hy_{t-1} = \delta_0(1 - h) + \delta_1(x_t - hx_{t-1}) + \zeta_t. \quad (25)$$

Notice that the autocorrelation in the true returns could introduce autocorrelation of the second order in the error terms. However, in an unreported investigation for the magnitude of this second order coefficient, we found that this coefficient is negligible: the average over the 72 stocks of the coefficient's lower bound is -0.017 and of its upper bound is -0.014. So, little efficiency is lost when we correct only for the first-order autocorrelation.

The slope estimate and its t -statistic equal:⁹

$$\hat{\delta}_1 = \frac{\text{cov}(y_t - hy_{t-1}, x_t - hx_{t-1})}{\text{var}(x_t - hx_{t-1})}, \quad (26)$$

$$\begin{aligned} t_{\hat{\delta}_1} &= \frac{\hat{\delta}_1 * \sqrt{n-2}}{\sqrt{\text{var}(\delta_1)}}, \\ &= \frac{\text{cov}(y_t - hy_{t-1}, x_t - hx_{t-1}) * \sqrt{n-2}}{\sqrt{\text{var}(x_t - hx_{t-1}) * \text{var}(y_t - hy_{t-1}) - (\text{cov}(y_t - hy_{t-1}, x_t - hx_{t-1}))^2}}, \end{aligned} \quad (27)$$

where n is the number of observations in the sum-of-return regression (22), which also equals to the number of observations in the regression $E_{t-\Delta}[r_{f,t} + r_{s,t+\tau}] = \gamma_0 + \gamma_1 p_{t-1}$.

In reality, the true morning return is unobserved, so the slope and $t_{\hat{\delta}_1}$ can not be estimated explicitly. Instead of estimating those coefficients directly, we will identify the sign of the slope and the lower and upper bounds of $t_{\hat{\delta}_1}$ in the next subsection. If the critical value of t -statistic 1.96 is outside the bounds, we can conclude on the significance of the sign of the slope δ_1 , and therefore on whether κ_s is significantly higher than κ_f or not.

⁸The first observation is omitted.

⁹In the classical linear regression, $y_t = \alpha + \beta x_t + e_t$, we have $\text{var}(e) = \text{var}(y) - \frac{(\text{cov}(x,y))^2}{\text{var}(x)}$, and $\text{var}(\beta) = \frac{\text{var}(e)}{(n-1) * \text{var}(x)}$, then $t\text{-stat} = \frac{\hat{\beta}}{\sqrt{\text{var}(\beta)}} = \frac{\text{cov}(x,y) * \sqrt{(n-1)}}{\sqrt{\text{var}(x) \text{var}(y) - (\text{cov}(x,y))^2}}$.

3.2 Sign of The Slope Estimate and Lower and Upper Bounds on $t_{\hat{\delta}_1}$

3.2.1 $|T\text{-stat}|$ as a Monotonically Increasing Function w.r.t. the Variance of the True Morning Return

In preparation for identifying the sign of the estimate $\hat{\delta}_1$ and the bounds of the $t_{\hat{\delta}_1}$, we rearrange the terms (26) and (27) using Assumptions (i)-(iii). Our objective is to express $\hat{\delta}_1$ and $t_{\hat{\delta}_1}$ as functions of the latent $\text{var}(\rho_t^m)$; in the next section we then derive bounds on this unobservable parameter so that we obtain bounds on the slope and its t -statistic.

The mathematical details are relegated to Appendix I, where it is shown that the slope estimate $\hat{\delta}_1$ becomes:

$$\hat{\delta}_1 = \frac{(1 + h + h^2)(\text{var}(e_{s,t+\tau}) - \text{var}(e_{f,t}))}{(1 + h^2)(\text{var}(p_t) - \text{var}(\rho_t^m))}, \quad (28)$$

$$= \frac{(1 + h + h^2)(1/2(\text{var}(r_{s,t+\tau}) - \text{var}(r_{f,t})))}{(1 + h^2)(\text{var}(p_t) - \text{var}(\rho_t^m))}. \quad (29)$$

Equation (28) implies that comparing the slopes κ_s and κ_f , *i.e.* identifying the sign of the $\hat{\delta}_1$, is equivalent to comparing the variance of the noise. Specifically, if the spot market is more noisy, *i.e.* $\text{var}(e_{s,t+\tau}) > \text{var}(e_{f,t})$ and thus $\hat{\delta}_1 > 0$, it becomes a follower in the price-discovery process. From Appendix I, we also get

$$t_{\hat{\delta}_1} = \frac{1/2(\text{var}(r_{s,t+\tau}) - \text{var}(r_{f,t}))\sqrt{n-2}}{\sqrt{\frac{((1+h^2)\text{var}(y_t) - 2h\text{COV}(y_t y_{t-1}))(1+h^2)}{(1+h+h^2)^2}(\text{var}(p_t) - \text{var}(\rho_t^m)) - \left(\frac{\text{var}(r_{s,t+\tau}) - \text{var}(r_{f,t})}{2}\right)^2}}, \quad (30)$$

where $h = h(\text{var}(\rho_t^m))$ is a function of $\text{var}(\rho_t^m)$ as in (A.6). According to formula (30), $t_{\hat{\delta}_1}$ is a function of $\text{var}(\rho_t^m)$. It is proven in Appendix II that $|t_{\hat{\delta}_1}|$ is a monotonically increasing function with respect to $\text{var}(\rho_t^m)$, *i.e.* increasing when $\text{var}(r_{s,t+\tau}) - \text{var}(r_{f,t}) > 0$ and decreasing when $\text{var}(r_{s,t+\tau}) - \text{var}(r_{f,t}) < 0$. So, depending on the bounds of the $\text{var}(\rho_t^m)$, which are presented in the next subsection, the bounds for $t_{\hat{\delta}_1}$ will be identified.

3.2.2 Bounds on The Variance of The True Morning Return and on the $t_{\hat{\delta}_1}$

We have eight second moments for observable returns, which, from Equations (14)-(16), can be expressed as functions of 10 underlying latent parameters:

$$\begin{aligned}\text{var}(r_{f,t}) &= \text{var}(\rho_t^m + \rho_t^r + \epsilon_{f,t} - \epsilon_{f,t-1}) \\ &= \text{var}(\rho_t^m + \rho_t^r) + 2\text{var}(\epsilon_{f,t})\end{aligned}\quad (31)$$

$$\begin{aligned}\text{cov}(r_{f,t}, r_{f,t-1}) &= \text{cov}(\rho_t^r + \rho_t^m + \epsilon_{f,t} - \epsilon_{f,t-1}, \rho_{t-1}^r + \rho_{t-1}^m + \epsilon_{f,t-1} - \epsilon_{f,t-2}) \\ &= \text{cov}(\rho_t^r + \rho_t^m, \rho_{t-1}^r + \rho_{t-1}^m) - \text{var}(\epsilon_{f,t})\end{aligned}\quad (32)$$

$$\begin{aligned}\text{var}(r_{s,t+\tau}) &= \text{var}(\rho_t^r + \rho_{t+1}^m + \epsilon_{s,t+\tau} - \epsilon_{f,t+\tau-1}) \\ &= \text{var}(\rho_t^r + \rho_{t+1}^m) + 2\text{var}(\epsilon_{s,t+\tau})\end{aligned}\quad (33)$$

$$\begin{aligned}\text{cov}(r_{s,t+\tau}, r_{s,t+\tau-1}) &= \text{cov}(\rho_t^r + \rho_{t+1}^m + \epsilon_{s,t+\tau} - \epsilon_{f,t+\tau-1}, \rho_{t-1}^r + \rho_t^m + \epsilon_{s,t+\tau-1} - \epsilon_{f,t+\tau-2}) \\ &= \text{cov}(\rho_t^r + \rho_{t+1}^m, \rho_{t-1}^r + \rho_t^m) - \text{var}(\epsilon_{s,t+\tau})\end{aligned}\quad (34)$$

$$\begin{aligned}\text{cov}(r_{f,t}, r_{s,t+\tau}) &= \text{cov}(\rho_t^m + \rho_t^r + \epsilon_{f,t} - \epsilon_{f,t-1}, \rho_t^r + \rho_{t+1}^m + \epsilon_{s,t+\tau} - \epsilon_{f,t+\tau-1}) \\ &= \text{cov}(\rho_t^m + \rho_t^r, \rho_t^r + \rho_{t+1}^m) + 2\text{cov}(\epsilon_{f,t}, \epsilon_{s,t+\tau})\end{aligned}\quad (35)$$

$$\begin{aligned}\text{cov}(r_{f,t}, r_{s,t+\tau-1}) &= \text{cov}(\rho_t^m + \rho_t^r + \epsilon_{f,t} - \epsilon_{f,t-1}, \rho_{t-1}^r + \rho_t^m + \epsilon_{s,t+\tau-1} - \epsilon_{f,t+\tau-2}) \\ &= \text{cov}(\rho_t^m + \rho_t^r, \rho_{t-1}^r + \rho_t^m) - \text{cov}(\epsilon_{f,t}, \epsilon_{s,t+\tau})\end{aligned}\quad (36)$$

$$\begin{aligned}\text{cov}(r_{f,t-1}, r_{s,t+\tau}) &= \text{cov}(\rho_{t-1}^r + \rho_{t-1}^m + \epsilon_{f,t-1} - \epsilon_{f,t-2}, \rho_t^r + \rho_{t+1}^m + \epsilon_{s,t+\tau} - \epsilon_{s,t+\tau-1}) \\ &= \text{cov}(\rho_{t-1}^r + \rho_{t-1}^m, \rho_t^r + \rho_{t+1}^m) - \text{cov}(\epsilon_{f,t}, \epsilon_{s,t+\tau})\end{aligned}\quad (37)$$

$$\text{cov}(p_t, p_{t-1}) = \text{cov}(\rho_t^m, \rho_{t-1}^m). \quad (38)$$

Below, we then extract the implications for the seven unobserved second moments for the morning and afternoon returns, expressed as functions of the observables:

$$\text{cov}(\rho_t^m, \rho_{t-1}^m) = \text{cov}(p_t, p_{t-1}). \quad (39)$$

$$\text{cov}(\rho_t^r, \rho_{t-1}^m) = \left(\frac{1}{2}\text{var}(r_{f,t}) + \text{cov}(r_{f,t}, r_{f,t-1})\right) - \left(\frac{1}{2}\text{var}(r_{s,t+\tau}) + \text{cov}(r_{s,t+\tau}, r_{s,t+\tau-1})\right). \quad (40)$$

$$\text{var}(\rho_t^m) + \text{cov}(\rho_t^r, \rho_t^m) + \text{cov}(\rho_{t-1}^r, \rho_t^m) = \text{cov}(r_{f,t}, r_{s,t+\tau-1}) - \text{cov}(r_{f,t-1}, r_{s,t+\tau}) + \text{cov}(\rho_t^r, \rho_{t-1}^m). \quad (41)$$

$$\begin{aligned}\text{var}(\rho_t^r) + \text{cov}(\rho_t^r, \rho_t^m) + \text{cov}(\rho_{t-1}^r, \rho_t^m) + 2\text{cov}(\rho_t^r, \rho_{t-1}^r) &= \text{cov}(r_{f,t}, r_{s,t+\tau}) + 2\text{cov}(r_{f,t-1}, r_{s,t+\tau}) - \\ &\quad - \text{cov}(p_t, p_{t-1}) - 2\text{cov}(\rho_t^r, \rho_{t-1}^m).\end{aligned}\quad (42)$$

To find bounds on the true morning return variance, we make two more assumptions:

(iv) The covariance of today's true morning and afternoon returns is nonnegative: $\text{cov}(\rho_t^m, \rho_t^r) \geq 0$.

(v) The variance of true morning return is not less than the variance of true afternoon return: $\text{var}(\rho_t^m) \geq \text{var}(\rho_t^r)$.

The last assumption is based on the fact that most trading happens in the morning, consistent with the idea that more information is released. As shown in Appendix III, we then find

$$\max\left(\frac{m}{3}, \frac{n}{5}\right) \leq \text{var}(\rho_t^m) \leq m, \quad (43)$$

where m and n are the right hand-side of Equations (41) and (42), respectively, which are numerically calculated from the data.

Notice from the relation (43) that the population parameter m is non-negative. However, for six (out of 72) stocks the sample estimate of m is negative. For these stocks, we can only use the (much wider) bound that the true morning return must be less variable than the observed daily returns, that is:

$$0 \leq \text{var}(\rho_t^m) \leq \text{var}(r_{f,t}), \quad (44)$$

These relations allow us to identify the upper and lower bounds, denoted as L and U respectively, of the $t_{\hat{\delta}_1}$. Specifically, when $\hat{\delta}_1$ is positive, *i.e.* the $t_{\hat{\delta}_1}$ is a monotonically increasing function w.r.t. $\text{var}(\rho_t^m)$, the lower bound L is computed at the minimum value of the $\text{var}(\rho_t^m)$, and the upper bound U is at the maximum value of $\text{var}(\rho_t^m)$. On the other hand, when $\hat{\delta}_1$ is negative, *i.e.* the function $t_{\hat{\delta}_1}$ monotonically decreases, the lower bound L is at the maximum and upper bound U is at the minimum value of $\text{var}(\rho_t^m)$. The formal formulas of these bounds on the $t_{\hat{\delta}_1}$ are presented in (A.42)-(A.48).

3.3 Results

We first check the autocorrelation in the forward premia. In the absence of synchronicity problems, the autocorrelation in the price discrepancies should be zero, see Equation (13); any autocorrelation would then be evidence of a lack of integration. This test becomes invalid when the two prices are not simultaneous and true returns are allowed to be autocorrelated: Equation (16) implies that

$$\frac{\text{cov}(p_t, p_{t-1})}{\text{var}(p_t)} = \frac{\text{cov}(\rho_t^m, \rho_{t-1}^m)}{\text{var}(\rho_t^m)}. \quad (45)$$

Table 7: Test of Autocorrelation in the Forward Premia

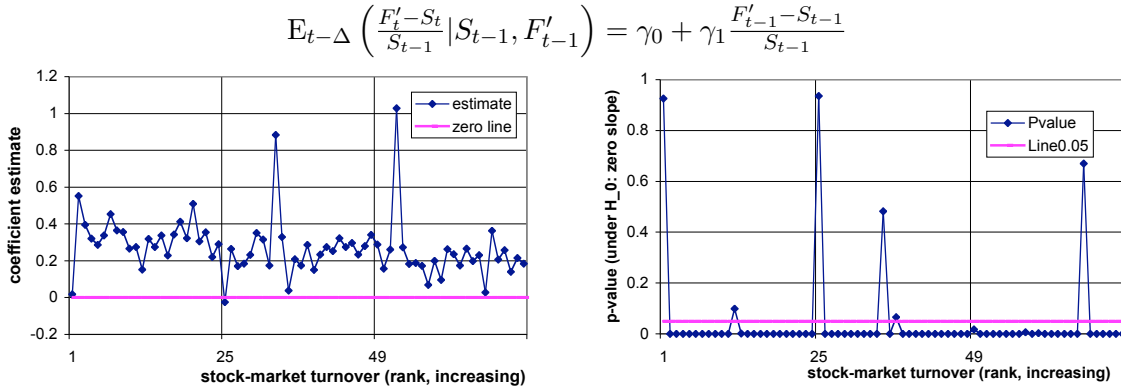
sample (by turnover)	$E_{t-\Delta} \left(\frac{F'_t - S_t}{S_{t-1}} S_{t-1}, F'_{t-1} \right) = \gamma_0 + \gamma_1 \frac{F'_{t+1} - S_{t-1}}{S_{t-1}}$					panel estimation			
	mean	median	$n_{>0}$	$\text{sgnf}_{>0}$	$\text{sgnf}_{<0}$	$\hat{\gamma}_1$	SE(.)	t-stat	prob
All	0.27	0.26	71	68	0	0.29	0.011	25.33	0.0000
Low turnover	0.32	0.32	24	23	0	0.32	0.022	14.30	0.0000
Medium	0.26	0.26	23	22	0	0.27	0.016	17.08	0.0000
High	0.24	0.20	24	23	0	0.26	0.012	21.50	0.0000

Key Ex post forward premia for 72 stocks are regressed on their lagged value. A zero γ means that forward premia are not correlated, a positive one signals positive autocorrelation in premia, meaning positive autocorrelation in the true morning return. We show summary statistics for all stocks and for three subsamples of stocks arranged by daily average turnover.

Figure 7 summarizes the individual stock estimates visually, while Table 7 provides some numerical information. The obvious feature is that autocorrelation is positive. Out of the total 72 cases, only one estimate actually is negative, and only marginally so, while 66 cases or 91.7% of the estimates are significantly positive. The averages and the number of significant rejections tend to fall the more active the stock is, but the effect is quite slight: the general average coefficient is 0.27, falling from 0.32 to 0.24 as we go from thinly to actively-traded stocks; the medians are similar.

For aggregates obtained via panel regression we test the independence assumption by regressing, for every equation, the 72 slopes on the corresponding turnovers. For the sample

Figure 7: Autocorrelation in the Forward Premia, Stock by Stock



Key Ex post forward premia for 72 stocks are regressed on their lagged value. This slope, γ_1 , estimates the scaled autocovariance of the forward premia. A zero γ_1 means that forward premia are not correlated, a positive one signals positive autocorrelation in premia, meaning that the true morning return is also autocorrelated of first-order. We show estimated gammas and their p-values for all stocks, arranged by daily average turnover.

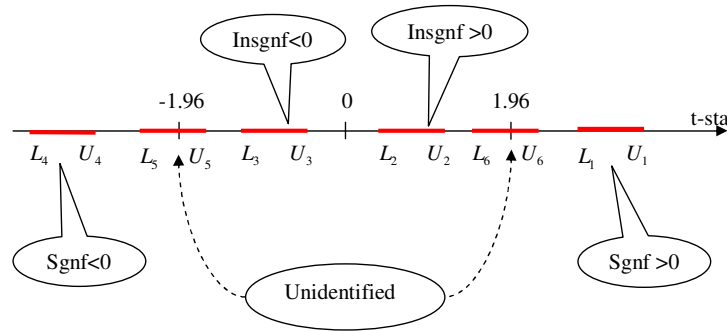
Table 8: **Preparing for Panel Estimation: Independence Tests for Slopes**

	regressing γ_j on turnover			regressing δ_j on turnover		
	slope	t-stat	prob	slope	t-stat	prob
All	-1.80	-2.63	0.0104	-15.10	-5.87	0.0000
Low turnover	2.96	0.10	0.9199	-142.38	-2.07	0.0505
Medium	12.35	1.90	0.0707	-84.63	-3.07	0.0057
High	-0.37	-0.40	0.6943	-2.97	-0.82	0.4207

Key γ_1 estimates the scaled autocovariance of forward premia. To be able to estimate the mean gamma via panel regressions with a common slope we need to test that individual stocks' gammas are deviating randomly from a general mean. Here we test whether there is a relation with turnover, first in the all-stock sample and then in the three subsamples of stocks assembled on the basis of average daily turnover.

as a whole there is, unsurprisingly, a significant negative relation, but within turnover groups there is no more clear link (Table 8). The aggregates are very similar to the straightforward means of individual estimates, and are clearly different from zero. All this implies that the true morning return is autocorrelated of at least first order, which is a sign of slow dissemination of fundamental information in at least one day. This phenomenon occurs across the entire spectrum of trading volume.

We now turn to the price-discovery issue. Recall that if we could regress the sum of the spot and forward return onto the beginning-of-period forward premium plus the true morning

Figure 8: **Six Possible Positions of the Critical Values -1.96 and 1.96 to the Range $[L \ U]$** 

Key Sgnf>0 is for significantly positive; Sgnf<0 is for significantly negative; Insgnf>0 is insignificantly positive; and Insgnf<0 is insignificantly negative. The crucial issue is whether 1.96 or -1.96 is within, or to the left, or to the right of the interval $[L \ U]$.

Table 9: Price Leadership Tests - Slope Estimate

$$E_{t-\Delta}[r_{f,t} + r_{s,t+\tau}] = \delta_0 + \delta_1(p_{t-1} + \rho_{t-1}^m).$$

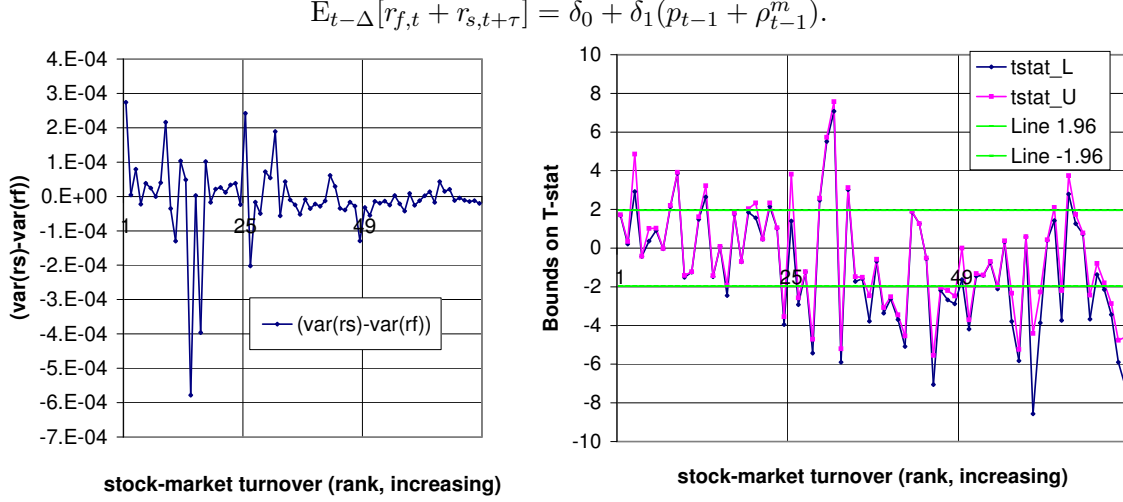
	Significant		Insignificant		Unidentified		All	
	Positive	Negative	Positive	Negative	Positive	Negative	Positive	Negative
All	10	23	16	15	4	4	30	42
Low	5	1	9	6	2	1	16	8
Medium	4	12	2	5	1	0	7	17
High	1	10	5	4	1	3	7	17

Key For 72 stocks, if the true morning return were observed, sums of spot and forward returns could be regressed on lagged forward premia plus the true morning return. δ_1 estimates $(\kappa_s - \kappa_f)$, where the κ s are the degrees to which a spot or forward price on average reacts to yesterday's forward premium plus the true morning return. A negative δ_1 means that the forward market adjusts less than the spot market. We report the summary statistics on the (in)significant positiveness/negativeness of the estimated deltas for all 72 stocks, arranged by daily average turnover.

return, the slope δ_1 provides an estimate of $\kappa_s - \kappa_f$, the difference of the correction speeds; see Equation (22). The interpretation is that if $\kappa_s - \kappa_f$ is positive, the spot market is more of a follower and the forward market more of a price discoverer, and vice versa. However, since the true morning return is unobserved, we cannot estimate the slope δ_1 explicitly. Instead, we can identify the sign of the slope δ_1 and the lower L and upper U bounds of the $t_{\hat{\delta}_1}$ according to formulas (A.8) and (A.41)-(A.48). The Appendix III reports the sign of the slope and the $t_{\hat{\delta}_1}$ bounds in detail. This information can also help to conclude whether the difference $\kappa_s - \kappa_f$ is significantly positive or significantly negative depending on the relative position of the critical value 1.96 or -1.96 to the range $[L \ U]$ of the $t_{\hat{\delta}_1}$, which is illustrated in Figure 8. Specifically, when $\hat{\delta}_1$ is positive, so is the $t_{\hat{\delta}_1}$: (i) if the critical value 1.96 is to the left of the range $[L \ U]$, then the slope δ_1 is significantly positive; (ii) if it is to the right of the range $[L \ U]$, then the slope δ_1 is insignificantly positive. On the other hand, when $\hat{\delta}_1$ is negative, so is $t_{\hat{\delta}_1}$: (i) if the critical value -1.96 is to the left of the range $[L \ U]$, then the slope δ_1 is insignificantly negative; (ii) if it is to the right of the range $[L \ U]$, then the slope δ_1 is significantly negative. In both cases of positive and negative $\hat{\delta}_1$, when the critical value 1.96 lies within the range $[L \ U]$, the significance of the slope δ_1 is unidentified in the context of our model.

Figure 9 reports the sign of the slopes and the bounds of the t-statistic of the price-discovery regression, stock-by-stock. In general, out of the total 72 cases, the estimate's significance is identified in 64 cases (89% of all the cases—21, 23, and 20 cases in the low-, medium-, and high-turnover groups correspondingly). Two observations stand out: (i) the slopes are negative in more cases in the total sample—that is, the forward market is more of a follower while the

Figure 9: **Sign of the Slopes and Bounds of the T-stat from the Price-Discovery Regression, Stock by Stock**



Key For 72 stocks, if the true morning return were observed, sums of spot and forward returns could be regressed on lagged forward premia plus the true morning return. δ_1 estimates $(\kappa_s - \kappa_f)$, where the κ s are the degrees to which a spot or forward price on average reacts to yesterday's forward premium plus the true morning return. A negative δ_1 means that the forward market adjusts less than the spot market. Since the true morning return is unobserved, we can only identify the sign of the slopes and estimate the bounds of t_{δ_1} . The right hand-side graph shows the lower and upper bounds of the t-stat of the δ_1 estimate. We show the results for all 72 stocks, arranged by daily average turnover.

spot market leads—and (ii) this is especially pronounced in the more active market sections. 42 estimates out of 72 are negative, and 23 significantly so. Notably, only in the low-turnover group, the δ_1 estimates are positive in more cases: out of the 21 estimates whose significance is determined by the bounds, 14 cases have positive slopes against 7 negative, and 5 positive cases are significant against only one negative. This result implies that the forward market is more of a leader in this market section. In the other two groups, the medium- and low-turnover samples, the spot market leads in 17 and 14 cases, respectively, of which 12 and 10 significantly so. To sum up, only in the low-activity group the forward market is more of a leader; in the two remaining market sections, in contrast, the price leadership status of the spot market is statistically evident in the majority of the cases. Thus, the evidence supports the conclusion from the exploratory variance comparisons: the spot market tends to be the price discoverer, the forward seems to be the one that lags behind.

4 Conclusions

Trading the same assets with different settlement rules in the BSE, the spot and forward tiers were supposed to have homogeneous expectations about the next opening price if they are integrated. Given this assumption, we could investigate the price-discovery issue in the two market tiers in the aspect of the prices' reaction to yesterday's forward premium. Since the opening prices of the spot and forward markets were asynchronous, the forward premium become the negative of the true morning return plus two pricing errors. Therefore, we adjust the ECMs: since prices should be correcting only the two lagged pricing errors, the ECMs should ideally be measuring the prices' reaction to yesterday's forward premium plus the true morning return. However, since the true morning return is unobserved, the correction speeds in our price-discovery model can not be estimated explicitly. Nevertheless, the (in)significant positiveness/negativeness of the difference of the correction speeds can still be identified on the basis of the bounds of $t_{\hat{\delta}_1}$. Though the forward market looked like the prime *a priori* candidate for the role of price discoverer, the empirical result show that the spot market leads in more cases.

From a wider perspective, in light of persistent pricing anomalies even within one Exchange, our research raises the issue of how far the financial markets perform their central function of price discovery and how far the conventional wisdom can be trusted (e.g. the more trading volume the less noise the observed price contains). We can only speculate on the reasons why the spot market did so unexpectedly well. The price limits that, in the eyes of the Exchange, protect spot investors against unexpected thinness in the book can hardly be the cause: the returns for days before and after the price limits were reached were purged from the sample because there was no price. The intervention of the *hoekmannen* in the spot market (to reduce imbalances in the book by adding trades for their own account) might have helped; yet such interventions should have been more frequent in the low-liquidity sections, while we see the superior performance of the spot prices most strongly in the high-liquidity end of the market. The apparent preference for cash, steering sales to the spot tier and purchases to the forward one, may explain the persistence of price discrepancies but does not tell us why the spot section still ends up as the more efficient market.

Appendix I: Deriving the slope and its t -statistic

To rearrange terms in the formulas of the slope estimate and its t -statistic, we use the following equations. The first equation we have is the difference of the variance of noise. From the decomposition of forward and spot returns in Equations (14) and (15) and Assumption (iii), $\text{cov}(\rho_t^m, \rho_t^r) = \text{cov}(\rho_{t-1}^r, \rho_t^m)$, we have:

$$\text{var}(r_{s,t+\tau}) - \text{var}(r_f) = 2(\text{var}(e_{s,t+\tau}) - \text{var}(e_{f,t})), \quad (\text{A.1})$$

which was the basis of our exploratory tests (Section 2). For the present purpose, this equation implies that even though the noise is unobserved, the difference of their variances can be calculated numerically within the assumptions in our model.

The second equation is for the variance of the regressor $\text{var}(x_t) = \text{var}(p_{t-1} + \rho_t^m)$. From the decomposition of forward premium in Equation (16), we have:

$$\begin{aligned} \text{var}(p_{t-1}) &= \text{var}(-\rho_t^m + e_{f,t-1} - e_{s,t+\tau-1}), \\ &= \text{var}(\rho_t^m) + \text{var}(e_{f,t-1} - e_{s,t+\tau-1}), \\ \Rightarrow \text{var}(x_t) = \text{var}(p_{t-1} + \rho_t^m) &= \text{var}(e_{f,t-1} - e_{f,t+\tau-1}), \\ &= \text{var}(p_{t-1}) - \text{var}(\rho_t^m), \\ &= \text{var}(p_t) - \text{var}(\rho_t^m). \end{aligned} \quad (\text{A.2})$$

Thirdly, rewrite the regressor and the regressed of the sum-of-return regression (23) as follows,

$$\begin{aligned} x_t &= e_{f,t-1} - e_{s,t+\tau-1}, \\ y_t &= 2\rho_t^r + \rho_t^m + \rho_{t+1}^m + e_{f,t} - e_{f,t-1} + e_{s,t+\tau} - e_{s,t+\tau-1}, \\ y_{t-1} &= 2\rho_{t-1}^r + \rho_{t-1}^m + \rho_t^m + e_{f,t-1} - e_{f,t-2} + e_{s,t+\tau-1} - e_{s,t+\tau-2}. \end{aligned}$$

Using Equation (A.1), we then obtain the formulas for the covariances of x and y :

$$\text{cov}(x_t, y_t) = \text{var}(e_{s,t+\tau}) - \text{var}(e_{f,t}) = 1/2(\text{var}(r_{s,t+\tau}) - \text{var}(r_f)), \quad (\text{A.3})$$

$$\text{cov}(x_t, y_{t-1}) = -\text{var}(e_{s,t+\tau}) + \text{var}(e_{f,t}) = -1/2(\text{var}(r_{s,t+\tau}) - \text{var}(r_f)), \quad (\text{A.4})$$

$$\text{cov}(x_{t-1}, y_t) = \text{cov}(x_t, x_{t-1}) = 0. \quad (\text{A.5})$$

Now we plug the terms in (A.1)-(A.5) into the formulas of the autocovariance coefficient h , the slope estimate $\hat{\delta}_1$, and the $t_{\hat{\delta}_1}$ in (24), (26), and (27), respectively. Notice that u_t is the

residual and $\hat{\delta}_0^*$, and $\hat{\delta}_1^*$ are the coefficient estimates of the OLS regression (23).

$$\begin{aligned}
\text{var}(\xi_t) &= \text{var}(y_t - \hat{\delta}_0^* - \hat{\delta}_1^* x_t) = \text{var}(y_t) - \hat{\delta}_1^* \text{cov}(y_t, x_t), \\
&= \text{var}(y_t) - \frac{(1/2(\text{var}(r_{s,t+\tau}) - \text{var}(r_{f,t})))^2}{(\text{var}(p_t) - \text{var}(\rho_t^m))}, \\
\text{cov}(\xi_t, \xi_{t-1}) &= \text{cov}(y_t - \hat{\delta}_0^* - \hat{\delta}_1^* x_t, y_{t-1} - \hat{\delta}_0^* - \hat{\delta}_1^* x_{t-1}) = \text{cov}(y_t, y_{t-1}) - \hat{\delta}_1^* \text{cov}(y_{t-1}, x_t), \\
&= \text{cov}(y_t, y_{t-1}) + \frac{(1/2(\text{var}(r_{s,t+\tau}) - \text{var}(r_{f,t})))^2}{(\text{var}(p_t) - \text{var}(\rho_t^m))}, \\
\Rightarrow h &= \frac{\text{cov}(\xi_t, \xi_{t-1})}{\text{var}(\xi_t)} = \frac{\text{cov}(y_t, y_{t-1}) + \frac{(1/2(\text{var}(r_{s,t+\tau}) - \text{var}(r_{f,t})))^2}{(\text{var}(p_t) - \text{var}(\rho_t^m))}}{\text{var}(y_t) - \frac{(1/2(\text{var}(r_{s,t+\tau}) - \text{var}(r_{f,t})))^2}{(\text{var}(p_t) - \text{var}(\rho_t^m))}}. \tag{A.6}
\end{aligned}$$

The slope estimate $\hat{\delta}_1$ and t -statistic then become:

$$\begin{aligned}
\hat{\delta}_1 &= \frac{\text{cov}(y_t - hy_{t-1}, x_t - hx_{t-1})}{\text{var}(x_t - hx_{t-1})} = \frac{(1 + h^2)\text{cov}(x_t y_t) - h\text{cov}(y_{t-1} x_t)}{(1 + h^2)\text{var}(x)}, \\
&= \frac{(1 + h + h^2)(\text{var}(e_{s,t+\tau}) - \text{var}(e_{f,t}))}{(1 + h^2)(\text{var}(p_t) - \text{var}(\rho_t^m))}, \tag{A.7}
\end{aligned}$$

$$= \frac{(1 + h + h^2)(1/2(\text{var}(r_{s,t+\tau}) - \text{var}(r_{f,t})))}{(1 + h^2)(\text{var}(p_t) - \text{var}(\rho_t^m))}. \tag{A.8}$$

Equation (A.7) implies that comparing the slopes κ_s and κ_f , *i.e.* identifying the sign of the $\hat{\delta}_1$, is equivalent to comparing the variances of the noise terms. Specifically, if the spot market is more noisy, *i.e.* $\text{var}(e_{s,t+\tau}) > \text{var}(e_{f,t})$ and thus $\hat{\delta}_1 > 0$, it becomes a follower in the price-discovery process.

The t -statistic obtained from (27) is implemented as follows:

$$\begin{aligned}
t_{\hat{\delta}_1} &= \frac{\text{cov}(y_t - hy_{t-1}, x_t - hx_{t-1})\sqrt{n-2}}{\sqrt{\text{var}(y_t - hy_{t-1}) * \text{var}(x_t - hx_{t-1}) - (\text{cov}(y_t - hy_{t-1}, x_t - hx_{t-1}))^2}}, \\
&= \frac{(1 + h + h^2)(1/2(\text{var}(r_{s,t+\tau}) - \text{var}(r_{f,t})))\sqrt{n-2}}{\sqrt{((1 + h^2)\text{var}(y_t) - 2h\text{cov}(y_t y_{t-1})) * (1 + h^2)\text{var}(x_t) - (1 + h + h^2)^2(1/2(\text{var}(r_{s,t+\tau}) - \text{var}(r_{f,t})))^2}}, \\
&= \frac{1/2(\text{var}(r_{s,t+\tau}) - \text{var}(r_{f,t}))\sqrt{n-2}}{\sqrt{\frac{((1 + h^2)\text{var}(y_t) - 2h\text{cov}(y_t y_{t-1}))(1 + h^2)}{(1 + h + h^2)^2}(\text{var}(p_t) - \text{var}(\rho_t^m)) - (1/2(\text{var}(r_{s,t+\tau}) - \text{var}(r_{f,t})))^2}}. \tag{A.9}
\end{aligned}$$

Appendix II: $|T\text{-stat}|$ as A Monotonically Increasing Function w.r.t. the Variance of the True Morning Return.

Denote:

$$a = \text{var}(y_t), \quad (\text{A.10})$$

$$b = \text{cov}(y_t, y_{t-1}),$$

$$c = \text{var}(p_t),$$

$$d = 1/2 * (\text{var}(r_{s,t+\tau}) - \text{var}(r_{f,t})),$$

$$z = \text{var}(\rho_t^m).$$

the autocorrelation coefficient h and the t -statistic in (A.6) and (A.9) correspondingly become:

$$h(z) = \frac{b + \frac{d^2}{c-z}}{a - \frac{d^2}{c-z}}, \quad (\text{A.11})$$

$$= \frac{b(c-z) + d^2}{a(c-z) - d^2}. \quad (\text{A.12})$$

$$t_{\hat{\delta}_1} = \frac{d\sqrt{n-2}}{\sqrt{\frac{((1+h^2)a-2hb)(1+h^2)}{(1+h+h^2)^2}(c-z) - d^2}}. \quad (\text{A.13})$$

Firstly, we show that $h(z)$ is a monotonically increasing function w.r.t. z , which is equivalent of showing its first derivative is positive.

$$h'_z = \frac{(a+b)d^2}{(a(c-z) - d^2)^2}. \quad (\text{A.14})$$

Since $a+b = \text{var}(y_t) + \text{cov}(y_t, y_{t-1}) > 0$, the first-order derivative h'_z is positive, and therefore $h(z)$ function monotonically increases w.r.t. z .

Next we show that the function $g(h) = \frac{((1+h^2)a-2hb)(1+h^2)}{(1+h+h^2)^2}$ monotonically decrease w.r.t. h . Denote $g_1(h) = \frac{(1+h^2)a-2hb}{1+h+h^2}$ and $g_2(h) = \frac{1+h^2}{1+h+h^2}$, we have:

$$g'_1(h) = \frac{(2ah-2b)(1+h+h^2) - (a(1+h^2) - 2hb)(2h+1)}{(1+h+h^2)^2}, \quad (\text{A.15})$$

$$= \frac{(h^2-1)(2b+a)}{(1+h+h^2)^2}. \quad (\text{A.16})$$

Since $h^2-1 = (\frac{\text{cov}(\xi_t, \xi_{t-1})}{\text{var}(\xi_t)})^2 - 1 < 0$ and $2b+a = 2\text{cov}(y_t, y_{t-1}) + \text{var}(y_t)$ is positive for all of the 72 stocks, the derivative $g'_1(h)$ is negative.

The derivative of the function $g_2(h) = \frac{1+h^2}{1+h+h^2}$ is:

$$g_2'(h) = \frac{2h(1+h+h^2) - (1+h^2)(2h+1)}{(1+h+h^2)^2}, \quad (\text{A.17})$$

$$= \frac{h^2 - 1}{(1+h+h^2)^2}. \quad (\text{A.18})$$

Also because $h^2 - 1 < 0$, the derivative $g_2'(h)$ is negative.

The derivative of $g(h) = g_1(h) * g_2(h)$ is:

$$g'(h) = g_1'(h) * g_2(h) + g_1(h) * g_2'(h). \quad (\text{A.19})$$

Since $g_1(h)$ and $g_2(h)$ are positive, we have that the derivative of the function $g(h)$ is negative. Taking derivative of $g(h(z))$ w.r.t. z , we have:

$$g_z' = g'(h) * h'(z), \quad (\text{A.20})$$

$$(\text{A.21})$$

Since $g'(h) < 0$ and $h'(z) > 0$, we have $g_z' < 0$. Therefore we have that the function $f(z) = \frac{((1+h^2)a-2hb)(1+h^2)}{(1+h+h^2)^2} * (c-z) = g(h(z)) * (c-z)$ has the negative first-order derivative w.r.t. z .

The $t_{\hat{\delta}_1}$ can be written as a function of z as follow:

$$t_{\hat{\delta}_1} = \frac{d\sqrt{n-2}}{\sqrt{f(z) - d^2}}. \quad (\text{A.22})$$

$$(\text{A.23})$$

The first derivative of the t -statistic is:

$$t\text{-stat}'_{\hat{\delta}_1} = \frac{-d\sqrt{n-2}}{2(\sqrt{f(z) - d^2})^3} * f'(z). \quad (\text{A.24})$$

$$(\text{A.25})$$

Since $f'(z)$ is negative, as shown above, we have that the $t\text{-stat}'_{\hat{\delta}_1}$ is positive when $d > 0$ and thus $t_{\hat{\delta}_1}$ monotonically increase w.r.t. z , and negative when $d < 0$ and thus $t_{\hat{\delta}_1}$ monotonically decrease w.r.t. z .

Appendix III: Bounds on the Variance of the True Morning Return and the $t_{\hat{\delta}_1}$

Applying the Assumptions (i) - (v) and using the decomposition of spot and forward returns and premium specified in (14) - (16), we have the following four equations, which are used to identify the bounds for the variance of the true morning return $\text{var}(\rho_t^m)$:

$$\text{cov}(\rho_t^m, \rho_{t-1}^m) = \text{cov}(p_t, p_{t-1}). \quad (\text{A.26})$$

$$\text{cov}(\rho_t^r, \rho_{t-1}^m) = \left(\frac{1}{2}\text{var}(r_{f,t}) + \text{cov}(r_{f,t}, r_{f,t-1})\right) - \left(\frac{1}{2}\text{var}(r_{s,t+\tau}) + \text{cov}(r_{s,t+\tau}, r_{s,t+\tau-1})\right). \quad (\text{A.27})$$

$$\text{var}(\rho_t^m) + \text{cov}(\rho_t^r, \rho_t^m) + \text{cov}(\rho_{t-1}^r, \rho_t^m) = \text{cov}(r_{f,t}, r_{s,t+\tau-1}) - \text{cov}(r_{f,t-1}, r_{s,t+\tau}) + \text{cov}(\rho_t^r, \rho_{t-1}^m). \quad (\text{A.28})$$

$$\begin{aligned} \text{var}(\rho_t^r) + \text{cov}(\rho_t^r, \rho_t^m) + \text{cov}(\rho_{t-1}^r, \rho_t^m) + 2\text{cov}(\rho_t^r, \rho_{t-1}^r) &= \text{cov}(r_{f,t}, r_{s,t+\tau}) + 2\text{cov}(r_{f,t-1}, r_{s,t+\tau}) - \\ &\quad - \text{cov}(p_t, p_{t-1}) - 2\text{cov}(\rho_t^r, \rho_{t-1}^m). \end{aligned} \quad (\text{A.29})$$

Now we use the above equations to identify the bounds of $\text{var}(\rho_t^m)$. For convenience, denote the right-hand-side of Equations (A.28) and (A.29) as m and n , respectively. The parameters m and n are numerically calculated from the data according to the following formulas:

$$\begin{aligned} m &= \text{cov}(r_{f,t}, r_{s,t+\tau-1}) - \text{cov}(r_{f,t-1}, r_{s,t+\tau}) + \left(\frac{1}{2}\text{var}(r_{f,t}) + \text{cov}(r_{f,t}, r_{f,t-1})\right) - \\ &\quad - \left(\frac{1}{2}\text{var}(r_{s,t+\tau}) + \text{cov}(r_{s,t+\tau}, r_{s,t+\tau-1})\right), \end{aligned} \quad (\text{A.30})$$

$$\begin{aligned} n &= \text{cov}(r_{f,t}, r_{s,t+\tau}) + 2\text{cov}(r_{f,t-1}, r_{s,t+\tau}) - \text{cov}(p_t, p_{t-1}) - (\text{var}(r_{f,t}) + 2\text{cov}(r_{f,t}, r_{f,t-1})) + \\ &\quad + (\text{var}(r_{s,t+\tau}) + 2\text{cov}(r_{s,t+\tau}, r_{s,t+\tau-1})). \end{aligned} \quad (\text{A.31})$$

So, we can rewrite Equations (A.28) and (A.29) as:

$$\text{var}(\rho_t^m) + \text{cov}(\rho_t^r, \rho_t^m) + \text{cov}(\rho_{t-1}^r, \rho_t^m) = m, \quad (\text{A.32})$$

$$\text{var}(\rho_t^r) + \text{cov}(\rho_t^r, \rho_t^m) + \text{cov}(\rho_{t-1}^r, \rho_t^m) + 2\text{cov}(\rho_t^r, \rho_{t-1}^r) = n. \quad (\text{A.33})$$

Making use of the Assumption (iii) and (iv), *i.e.* $\text{cov}(\rho_t^r, \rho_t^m) = \text{cov}(\rho_{t-1}^r, \rho_t^m) \geq 0$, and the Assumption (v), *i.e.* $\text{var}(\rho_t^r) \leq \text{var}(\rho_t^m)$, we have¹⁰:

$$0 \leq \text{cov}(\rho_t^r, \rho_t^m) \leq \text{var}(\rho_t^m), \quad (\text{A.34})$$

$$0 \leq \text{cov}(\rho_{t-1}^r, \rho_t^m) \leq \text{var}(\rho_t^m), \quad (\text{A.35})$$

$$\text{cov}(\rho_t^r, \rho_{t-1}^r) \leq \text{var}(\rho_t^m). \quad (\text{A.36})$$

Applying these inequalities for Equations (A.32) and (A.33), we obtain the following bounds:

$$\text{var}(\rho_t^m) \leq \text{var}(\rho_t^m) + \text{cov}(\rho_t^r, \rho_t^m) + \text{cov}(\rho_{t-1}^r, \rho_t^m) = m \leq 3\text{var}(\rho_t^m), \quad (\text{A.37})$$

$$\text{var}(\rho_t^r) + \text{cov}(\rho_t^r, \rho_t^m) + \text{cov}(\rho_{t-1}^r, \rho_t^m) + 2\text{cov}(\rho_t^r, \rho_{t-1}^r) = n \leq 5\text{var}(\rho_t^m). \quad (\text{A.38})$$

¹⁰Proof: $\text{cov}(\rho_t^r, \rho_t^m) = \sqrt{\text{var}(\rho_t^r)\text{var}(\rho_t^m)}\text{cor}(\rho_t^r, \rho_t^m) \leq \text{var}(\rho_t^m)$.

So, the bounds for $\text{var}(\rho_t^m)$ is:

$$\max\left(\frac{m}{3}, \frac{n}{5}\right) \leq \text{var}(\rho_t^m) \leq m, \quad (\text{A.39})$$

Notice from the relation (A.39) that the population parameter m is non-negative. However, for six (out of 72) stocks the sample estimate of m is negative. For these stocks, we can only use the (much wider) bound that the true morning return must be less variable than the observed daily returns, that is:

$$0 \leq \text{var}(\rho_t^m) \leq \text{var}(r_{f,t}), \quad (\text{A.40})$$

So, bounds for $\text{var}(\rho_t^m)$ is (A.39) in the cases of $m > 0$ and (A.40) in the cases of $m < 0$.

Now we turn to the bounds of $t_{\hat{\delta}_1}$. As shown in the previous subsection, the $|t_{\hat{\delta}_1}|$ is a monotonically increasing function with respect to $\text{var}(\rho_t^m)$. Therefore, the bounds for the $t_{\hat{\delta}_1}$ are:

(★) Cases of $m > 0$, the bounds in (A.39) are used:

when $\text{var}(r_{s,t+\tau}) - \text{var}(r_f) > 0$:

$$L_{\hat{\delta}_1 > 0} = t_{\hat{\delta}_1}(\text{var}(\rho_t^m) = \max(\frac{m}{3}, \frac{n}{5})), \quad (\text{A.41})$$

$$U_{\hat{\delta}_1 > 0} = t_{\hat{\delta}_1}(\text{var}(\rho_t^m) = m). \quad (\text{A.42})$$

when $\text{var}(r_{s,t+\tau}) - \text{var}(r_f) < 0$:

$$L_{\hat{\delta}_1 < 0} = t_{\hat{\delta}_1}(\text{var}(\rho_t^m) = m), \quad (\text{A.43})$$

$$U_{\hat{\delta}_1 < 0} = t_{\hat{\delta}_1}(\text{var}(\rho_t^m) = \max(\frac{m}{3}, \frac{n}{5})). \quad (\text{A.44})$$

(★) Cases of $m < 0$, the bounds in (A.40) are used:

when $\text{var}(r_{s,t+\tau}) - \text{var}(r_f) > 0$:

$$L_{\hat{\delta}_1 > 0} = t_{\hat{\delta}_1}(\text{var}(\rho_t^m) = 0), \quad (\text{A.45})$$

$$U_{\hat{\delta}_1 > 0} = t_{\hat{\delta}_1}(\text{var}(\rho_t^m) = \text{var}(r_{f,t})). \quad (\text{A.46})$$

when $\text{var}(r_{s,t+\tau}) - \text{var}(r_f) < 0$:

$$L_{\hat{\delta}_1 < 0} = t_{\hat{\delta}_1}(\text{var}(\rho_t^m) = \text{var}(r_{f,t})), \quad (\text{A.47})$$

$$U_{\hat{\delta}_1 < 0} = t_{\hat{\delta}_1}(\text{var}(\rho_t^m) = 0). \quad (\text{A.48})$$

Appendix IV: Sign of the $\hat{\delta}_1$ and the Bounds of the T -statistic

Table 10: Sign of the $\hat{\delta}_1$ and the Bounds of T -statistic

Low				Medium				High			
id*	var(rs)- var(rf)	t-stat L U		id*	var(rs)- var(rf)	t-stat L U		id*	var(rs)- var(rf)	t-stat L U	
20	2.74E-04	1.69	1.72	294	2.42E-04	1.40	3.82	25	-3.24E-05	-1.63	
82	4.76E-06	0.22	0.35	15	-2.02E-04	-2.93	-2.57	40	-5.50E-05	-4.18	-3.70
11	7.96E-05	2.92	4.86	287	-1.65E-05	-1.23	-1.22	57	-1.42E-05	-1.44	-1.31
21	-2.24E-05	-0.43	-0.41	268	-4.94E-05	-5.43	-4.71	68	-1.94E-05	-1.42	-1.40
269	3.86E-05	0.36	1.01	52	7.22E-05	2.47	2.59	35	-1.35E-05	-0.75	-0.70
24	2.44E-05	0.91	1.03	9	5.38E-05	5.51	5.73	34	-2.50E-05	-2.11	-1.96
56	-4.84E-07	-0.03	-0.03	36	1.89E-04	7.08	7.56	30	3.00E-06	0.32	0.38
8	4.00E-05	2.14	2.20	260	-5.64E-05	-5.90	-5.20	13	-2.12E-05	-3.79	-2.33
23	2.16E-04	3.86	3.91	66	4.34E-05	3.02	3.13	39	-4.24E-05	-5.83	-5.24
293	-3.54E-05	-1.51	-1.41	76	-9.68E-06	-1.72	-1.47	61	9.10E-06	0.58	0.59
7	-1.30E-04	-1.24	-1.23	290	-2.44E-05	-1.59	-1.51	67	-2.54E-05	-8.57	-4.42
32	1.03E-04	1.49	1.62	71	-5.18E-05	-3.78	-2.47	46	-1.22E-05	-3.87	-2.27
77	4.88E-05	2.65	3.23	10	-8.48E-06	-0.68	-0.57	78	2.42E-06	0.41	0.43
55	-5.78E-04	-1.48	-1.43	72	-3.50E-05	-3.37	-3.07	19	1.34E-05	1.43	2.09
70	2.52E-06	0.08	0.08	29	-2.18E-05	-2.62	-2.52	1	-1.69E-05	-3.74	-2.19
16	-3.96E-04	-2.45	-1.91	60	-2.88E-05	-3.69	-3.45	65	4.32E-05	2.80	3.75
31	1.01E-04	1.76	1.80	289	-1.26E-05	-5.08	-4.54	59	1.50E-05	1.26	1.73
73	-1.70E-05	-0.71	-0.70	5	6.14E-05	1.86	1.89	47	2.08E-05	0.73	0.78
291	2.12E-05	1.85	2.04	48	2.92E-05	1.26	1.26	43	-1.14E-05	-3.67	-2.43
51	2.64E-05	1.56	2.33	26	-3.46E-05	-0.55	-0.52	69	-4.52E-06	-1.37	-0.80
296	1.17E-05	0.46	0.47	58	-3.94E-05	-7.06	-5.54	27	-1.13E-05	-2.14	-1.79
53	3.36E-05	2.15	2.33	64	-1.65E-05	-2.19	-2.03	41	-1.47E-05	-3.44	-2.87
62	3.82E-05	1.03	1.05	37	-2.82E-05	-2.67	-2.20	63	-1.25E-05	-5.90	-4.77
292	-2.40E-05	-3.95	-3.54	17	-1.29E-04	-2.88	-2.48	42	-2.02E-05	-7.19	-4.60

Key: id* is the stock id.

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